

Addition and Subtraction of Polynomials

Polynomials with *exactly* the same variable combinations can be added or subtracted. For example, $7xy$ and $3xy$ have the same variable combination. We call these **like terms**.

$$7xy + 3xy = 10xy \quad \text{and} \quad 7xy - 3xy = 4xy$$

A polynomial is in *simplest form* if it contains no **grouping symbols** (except a *fraction bar*) and all *like terms* have been combined.

Polynomials can be arranged in any order. In **standard form**, polynomials are arranged from *left to right*, from *greatest to least* degree of *power*. For example:

$$x^7 - x^2 + 8x$$

Polynomials can be added or subtracted in vertical ($\updownarrow$) or horizontal ($\leftrightarrow$) form.

Addition

vertical form

$$(3y^2 + 2y + 3) + (y^2 + 1)$$

Align like terms in columns and add.

$$\begin{array}{r} 3y^2 + 2y + 3 \\ (+) \quad y^2 \quad + 1 \\ \hline 4y^2 + 2y + 4 \end{array}$$

write degrees of powers
left to right from greatest
to least

align like terms

add like terms

horizontal form

$$(3y^2 + 2y + 3) + (y^2 + 1)$$

Regroup and add like terms.

$$(3y^2 + y^2) + (2y) + (3 + 1) =$$

group like terms

$$4y^2 + 2y + 4$$

add like terms

Subtraction

You subtract a polynomial by adding its **additive inverse** or **opposite**. To do this, multiply each term in the *subtracted* polynomial by -1 and add.

polynomial	additive inverse
$-8y + 4x$	$8y - 4x$
$3q^2 - 6r + 11$	$-3q^2 + 6r - 11$
$2a + 7b - 3$	$-2a - 7b + 3$

vertical form

$$(3y^2 - 2y + 3) - (y^2 - 1)$$

Align like terms in columns and subtract by adding the additive inverse.

	write degrees of powers left to right from greatest to least	
$3y^2 - 2y + 3$	↙ ↘	$3y^2 - 2y + 3$
$(-) \quad y^2 \quad - 1$	align like terms	$(+) \quad -y^2 \quad + 1$
	add additive inverse	
	add like terms	$2y^2 - 2y + 4$



Remember: $-y^2 = -1y^2$

horizontal form

$$(3y^2 - 2y + 3) - (y^2 - 1)$$

Subtract by adding *additive inverse* and group like terms.

$$[3y^2 + (-2y) + 3] + [(-y^2) + 1] = \leftarrow \begin{array}{l} \text{add additive inverse of } 2y, \text{ which} \\ \text{is } -2y, \text{ and } y^2 - 1, \text{ which is } -y^2 + 1 \end{array}$$

$$[3y^2 + (-y^2)] + (-2y) + (3 + 1) = \leftarrow \text{group like terms}$$

$$2y^2 + -2y + 4 = \leftarrow \text{add like terms}$$

vertical form

Subtract $2t^2 - 3t + 4$ from the **sum** of $t^2 + t - 6$ and $3t^2 + 2t - 1$.

$$(t^2 + t - 6) + (3t^2 + 2t - 1) - (2t^2 - 3t + 4)$$

		write degrees of powers left to right from greatest to least		
$t^2 + t - 6$	←		→	$t^2 + t - 6$
$3t^2 + 2t - 1$	←	align like terms	→	$3t^2 + 2t - 1$
$(-) \underline{2t^2 - 3t + 4}$	←	add additive inverse	→	$(+) \underline{-2t^2 + 3t - 4}$
				$2t^2 + 6t - 11$

horizontal form

Subtract $2t^2 - 3t + 4$ from the *sum* of $t^2 + t - 6$ and $3t^2 + 2t - 1$.

$$(t^2 + t - 6) + (3t^2 + 2t - 1) - (2t^2 - 3t + 4)$$

$$\boxed{t^2} + \boxed{t} - \boxed{6} + \boxed{3t^2} + \boxed{2t} - \boxed{1} - \boxed{2t^2} + \boxed{3t} - \boxed{4} = \leftarrow \begin{array}{l} \text{add additive inverse of} \\ -(2t^2 - 3t + 4), \text{ which is} \\ -2t^2 + 3t - 4 \end{array}$$

$$(t^2 + 3t^2 - 2t^2) + (t + 2t + 3t) + (-6 - 1 - 4) = \leftarrow \text{group like terms}$$

$$2t^2 + 6t - 11 \leftarrow \text{combine like terms}$$